CHAPTER 4 ❑ MATLAB EXERCISES

1. Let $\mathbf{u}_1 = (1, 1, 2, 2)$, $\mathbf{u}_2 = (2, 3, 5, 6)$, and $\mathbf{u}_3 = (2, -1, 3, 6)$. Use MATLAB to write (if possible) the vector $\mathbf{v}$ as a linear combination of the vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$.

(a) $\mathbf{v} = (0, 5, 3, 0)$

(b) $\mathbf{v} = (-1, 6, 1, -4)$

2. Use MATLAB to determine whether the given set of vectors spans $\mathbf{R}^4$.

(a) $\{(1, -2, 3, 4), (2, 4, 5, 0), (-2, 0, 0, 4), (3, 2, 1, -4)\}$

(b) $\{(0, 1, -1, 1), (2, -2, 3, 1), (7, 0, 1, 0), (5, 2, -2, -1)\}$

3. Use MATLAB to determine whether the set is linearly independent or dependent.

(a) $\{(0, 1, -3, 4), (-1, 0, 0, 2), (0, 5, 3, 0), (-1, 7, -3, -6)\}$

(b) $\{(0, 0, 1, 2, 3), (0, 0, 2, 3, 1), (1, 2, 3, 4, 5), (2, 1, 0, 0, 0), (-1, -3, -5, 0, 0)\}$

4. Use MATLAB to determine whether the set of vectors forms a basis of $\mathbf{R}^4$.

(a) $\{(1, -2, 3, 4), (2, 4, 5, 0), (-2, 0, 0, 4), (3, 2, 1, -4)\}$

(b) $\{(0, 1, -1, 1), (2, -2, 3, 1), (7, 0, 1, 0), (5, 2, -2, -1)\}$

(c) $\{(0, 1, -3, 4), (-1, 0, 0, 2), (0, 5, 3, 0), (-1, 7, -3, -6)\}$

(d) $\{(0, 0, 1, 2), (0, 2, 3, 1), (1, 3, 4, 5), (2, 1, 0, 0), (-3, -5, 0, 0)\}$

5. Suppose you want to find a basis for $\mathbf{R}^4$ that contains the vectors $\mathbf{v}_1 = (1, 1, 0, 0)$ and $\mathbf{v}_2 = (1, 0, 1, 0)$. One way to do this is to consider the set of vectors consisting of $\mathbf{v}_1$ and $\mathbf{v}_2$, together with the standard basis vectors, $\mathbf{e}_1 = (1, 0, 0, 0)$, $\mathbf{e}_2 = (0, 1, 0, 0)$, $\mathbf{e}_3 = (0, 0, 1, 0)$, and $\mathbf{e}_4 = (0, 0, 0, 1)$. Let A be the matrix whose columns consist of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$, and $\mathbf{e}_4$, and apply **rref** to A to obtain

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Since the leading ones of the reduced matrix on the right are in columns 1, 2, 3, and 6, a basis for $\mathbf{R}^4$ consists of the corresponding column vectors of A: $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_1, \mathbf{e}_4\}$.

A convenient way to construct the matrix A is to define the matrix B whose columns are the given vectors $\mathbf{v}_1$ and $\mathbf{v}_2$. Then A is simply the matrix obtained by adjoining the 4×4 identity matrix to B: $A = [B \quad \text{eye}(4)]$.

Use this algorithm to find a basis for $\mathbf{R}^5$ that contains the given vectors.

(a) $\mathbf{v}_1 = (2, 1, 0, 0, 0)$, $\quad \mathbf{v}_2 = (-1, 0, 1, 0, 0)$,

(b) $\mathbf{v}_1 = (1, 0, 2, 0, 0)$, $\quad \mathbf{v}_2 = (1, 1, 2, 0, 0)$, $\quad \mathbf{v}_3 = (1, 1, 1, 0, 1)$

6. Use MATLAB to find a subset of the given set of vectors that forms a basis for the span of the vectors.

(a) $\{(1, 2, -1, 0), (-3, -6, 3, 0), (1, 0, 0, 1), (-2, -2, 1, -1)\}$

(b) $\{(0, 0, 1, 1, 0), (1, 1, 0, 0, 1), (1, 1, 1, 1, 1), (1, 1, 2, 2, 1), (0, 0, 3, 3, 1), (0, 0, 0, 0, 1)\}$

7. Let

$$A = \begin{bmatrix} -1 & 2 & 0 & 0 & 3 \\ 0 & 2 & 3 & -1 & 2 \\ -1 & 4 & 3 & -1 & 5 \\ 2 & -4 & 0 & 0 & -6 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

(c) Use the MATLAB command **rank** to find the rank of A.

8. Find a basis for the nullspace of the given matrix A. Then verify that the sum of the rank and nullity of A equals the number of columns.

(a) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

(b) $A = \text{hilb}(5)$

(c) $A = \text{pascal}(5)$

(d) $A = \text{magic}(6)$

9. Let $\{(1, 0, 1),\ (0, -1, 2),\ (2, 3, -5)\}$ be a (nonstandard) basis for $\mathbf{R}^3$. You can find the coordinate matrix of $\mathbf{x} = (1, 2, -1)$ relative to this basis by writing $\mathbf{x}$ as a linear combination of the basis vectors. That is, the coordinate matrix is the solution vector to the linear system $B\mathbf{c} = \mathbf{x}$, where the basis vectors form the columns of B. Use MATLAB to solve this system and compare your answer to Section 4.7, Example 3.

10. Let $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $B1 = \{(1, 0, 1), (0, -1, 2), (2, 3, -5)\}$ be the two bases of R^3 given in Section 4.7, Example 4. We can use MATLAB to find the transition matrix from B to $B1$ by first forming the two matrices

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B1 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \\ 1 & 2 & -5 \end{bmatrix}.$$

Adjoin B and $B1$ by using the MATLAB command **C = [B1 B].** Let A be the reduced row-echelon form of C, **A = rref(C).** Finally, $PINV = P^{-1}$ is obtained by deleting the first three columns of this reduced matrix using the MATLAB command **PINV = A(:,4:6).** You obtain P simply by inverting $PINV$.

Find the transition matrix from B to $B1$.

(a) $B = \{(-3, 2), (4, -2)\}, \quad B1 = \{(-1, 2), (2, -2)\}.$

(b) $B = \{(1, 1, 1, 1), (0, 1, 1, 1), (0, 0, 1, 1), (0, 0, 0, 1)\},$
$B1 = \{(1, 0, 1, 0), (1, 0, -1, 0), (0, 1, 0, 1), (0, 1, 0, -1)\}$